Modulation and detection schemes

Outline

- Classical modulation techniques
- Frequency Shift Keying (FSK)
- Phase Shift Keying (PSK)
- Quadrature Phase Shift Keying (QPSK)
- Minimum Shift Keying (MSK)
- Analysis of bit error rates (BER)
- Power and spectral efficiency
- Conclusion

(FSK and PSK: Binary; QPSK and MSK: M-ary (multiple value))

• Classical modulation techniques

- Amplitude modulation

$$s_{AM}(t) = \left(A_0 + k_a e_m(t)\right)\cdot \cos\omega_0 t$$

AM signal is sensitive to noise and distortions

$$S_{AM}(\omega) = A_0\pi\left(\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right) + \frac{k_a}{2}\left(E_m(\omega-\omega_0)+E_m(\omega+\omega_0)\right)$$

- Frequency modulation

$$f(t) = f_0 + k_f e_m(t)$$

$$s_{FM}(t) = A_0 \cos\left(2\pi f_0 t + k_f \int e_m(t)dt\right)$$

Frequency deviation ($k_f e_m(t)$)

- Phase modulation

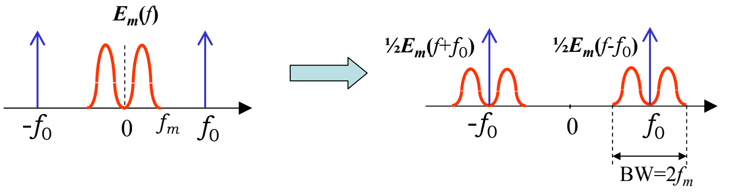
$$\theta(t) = \theta_0 + k_p e_m(t)$$

$$s_{PM}(t) = A_0 \cos\left(\omega_0 t + \theta_0 + k_p e_m(t)\right)$$

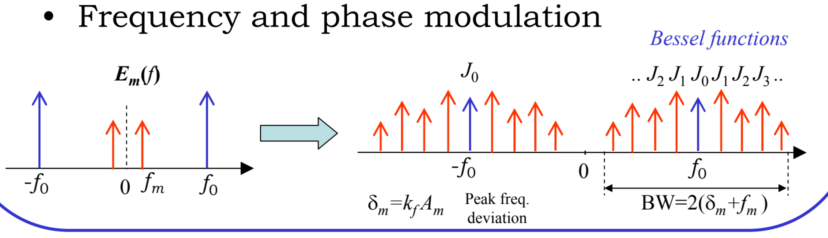
Phase deviation ($k_p e_m(t)$)

• Classical modulation techniques (cont'd)

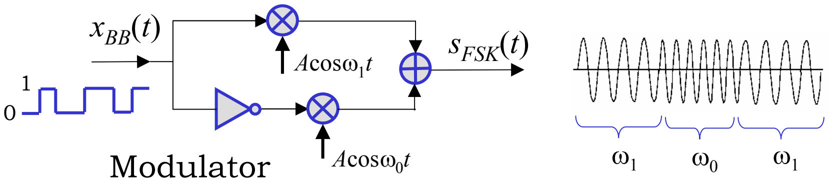
- Amplitude modulation



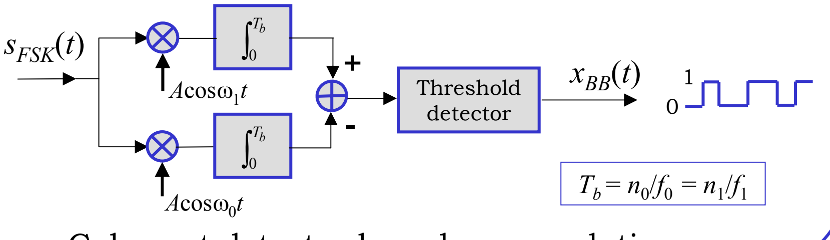
- Frequency and phase modulation



• Frequency Shift Keying (FSK)



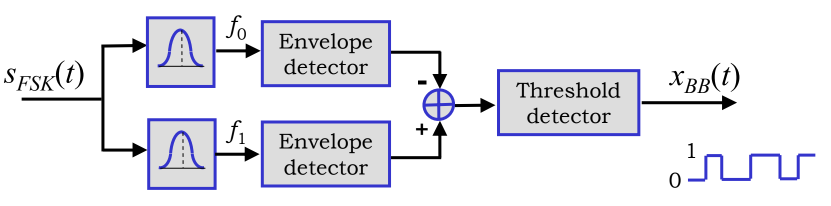
Modulator



Coherent detector based on correlation

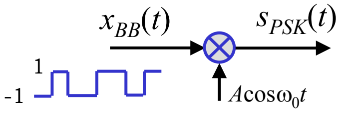
• **Frequency Shift Keying** (cont'd)

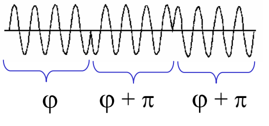
In coherent FSK detection oscillator and carrier need synchronization
When off-phase by $\pi/2$ the correlator outputs 0 instead of 1



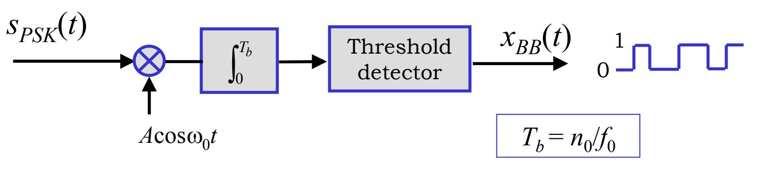
Non-coherent FSK detector (simpler receiver)

• **Phase Shift Keying (PSK)**





Modulator

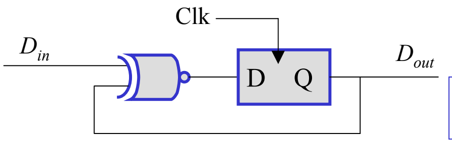


Coherent detector based on correlation
(non-coherent PSK detection possible only in differential mode)

• **Phase Shift Keying** (cont'd)

Differential Phase Shift Keying (DPSK)

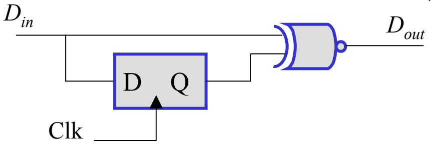
$$D_{out}((n+1)T_b) = \overline{D_{out}(nT_b) \oplus D_{in}(nT_b)}$$



Differential encoding
If input 1 then output unchanged, starting bit needed

In: - 0 1 1 1 0 0 1 1 0
Out: 1 0 0 0 0 1 0 0 0 1

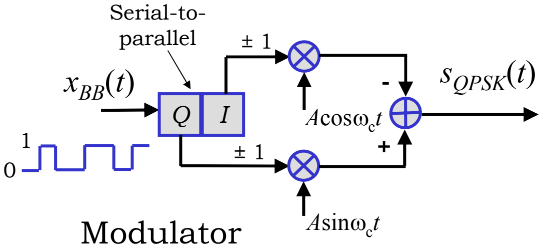
$$D_{out}(nT_b) = \overline{D_{in}((n+1)T_b) \oplus D_{in}(nT_b)}$$



Differential decoding
(non-coherent PSK detection)

More error prone than coherent (higher BER, but much simpler circuit)

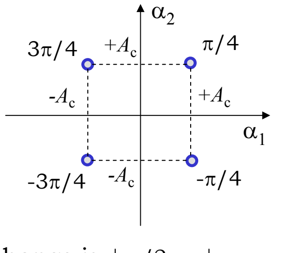
• **Quadrature Phase Shift Keying (QPSK)**



Modulator

Output signal takes on 4 values due to phases 0, π in one PSK channel, and π/2, 3π/2 in the other.

This happens every second input bit.



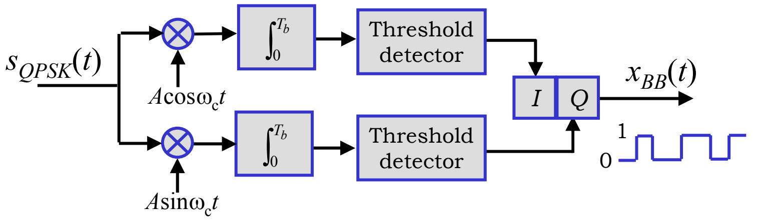
Model (constellation):

$$s_{QPSK}(t) = \alpha_1\cos\omega_c t + \alpha_2\sin\omega_c t$$

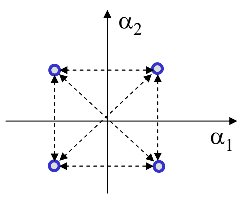
$$\alpha_{1,2} = \pm A_c$$

During transitions the phase change is ± π/2 or ± π

• **Quadrature Phase Shift Keying** (cont'd)



QPSK detector

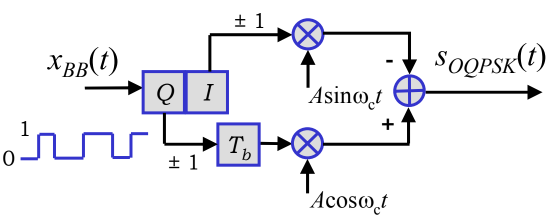


Phase transitions in QPSK

Advantage of QPSK:
As bits are grouped and transmitted in pairs, the bandwidth needed is half compared to binary PSK.

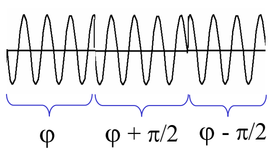
• **Quadrature Phase Shift Keying** (cont'd)

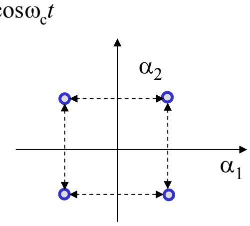
Offset QPSK modulation



Due to delay by 1 bit we avoid simultaneous transitions of bits in both branches

Advantage: all phase changes at output $\pm\pi/2$, narrower bandwidth needed, less demands on linearity of PA



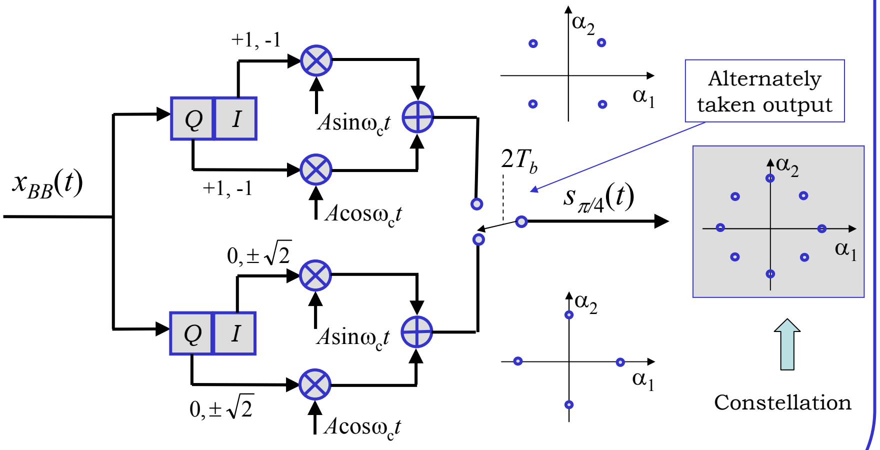


Phase transitions in OQPSK

Drawback: cannot be adopted to differential encoding to support non-coherent reception

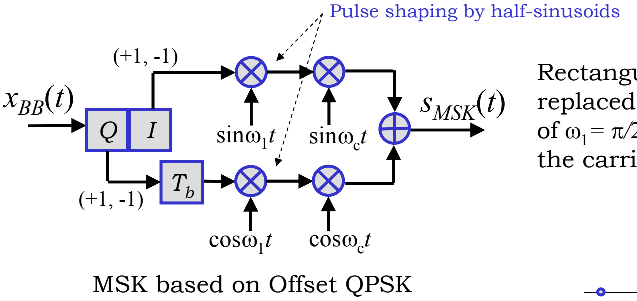
• **Quadrature Phase Shift Keying** (cont'd)

$\pi/4$ - QPSK modulation



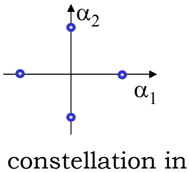
Advantage: $\pi/4$-QPSK undergoes differential encoding, useful for non-coherent reception, however max. phase change is $3\pi/4$

• **Minimum Shift Keying (MSK)**



MSK based on Offset QPSK

Rectangular pulses are replaced by half-sinusoids of $\omega_1 = \pi/2T_b$ that modulates the carrier of ω_c



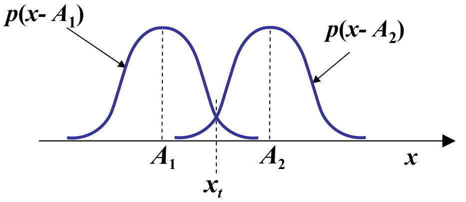
Phase constellation in MSK

Advantage: no abrupt phase changes at the output, signal bandwidth even more narrower !

Different variants of MSK exist, GMSK, GFSK, …

• Analysis of bit error rates (BER) in detection schemes

BER ~ P_e (probability of making an error when transmitting a symbol due to AWGN)



A_1, A_2 correct values detected,

Error when x_t crossed

P_{e1} - error in detecting A_1

P_e – total error in detection

$$P_{e1} = \frac{1}{2}\int_{x_t}^{\infty} p(x - A_1)dx$$

$$P_e = 2P_{e1} = \int_{x_t}^{\infty} p(x - A_1)dx = \ldots \quad = \mathrm{erfc}\left(\frac{1}{2}\sqrt{SNR_{max}}\right)$$

• Analysis of bit error rates (cont'd)

$$SNR_{max} = \frac{E_d}{N_0/2}$$

Power of AWGN

$$E_d = \int_0^{T_b} \left|p_1(t) - p_2(t)\right|^2 dx$$

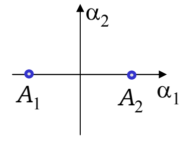
Energy of the modulated carrier

BPSK

$$p_1(t) = -p_2(t) = A_c \cos\omega_c t$$

$$E_d = \int_0^{T_b} \left|2A_c \cos\omega_c t\right|^2 dt = 2A_c^2 T_b$$

$$P_e = \mathrm{erfc}\left(\frac{1}{2}\sqrt{SNR_{max}}\right) = \mathrm{erfc}\left(\sqrt{\frac{A_c^2 T_b}{N_0}}\right)$$



Constellation for BPSK

Note that erfc() is a descending function

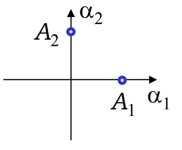
• **Analysis of bit error rates** (cont'd)

BFSK

$$p_1(t) = A_c \cos\omega_1 t, \quad p_2(t) = A_c \cos\omega_2 t$$

$$E_d = \int_0^{T_b} \left| p_1(t) - p_2(t) \right|^2 dt = A_c^2 T_b$$

$$P_e = \mathrm{erfc}\left(\frac{1}{2}\sqrt{SNR_{\max}} \right) = \mathrm{erfc}\left(\sqrt{\frac{A_c^2 T_b}{2N_0}} \right)$$



Constellation for BFSK
Distance between A1A2
is smaller than for BPSK

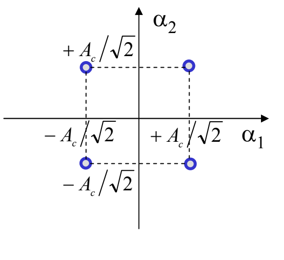
BER of BFSK is higher than for BPSK,

Compare distance A_1A_2 in their constellations.

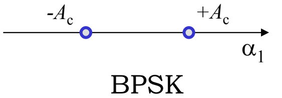
• **Analysis of bit error rates** (cont'd)

QPSK techniques

Scaled for equal average power per bit with BPSK



The respective points are closer to each other than in BPSK (smaller E_d), but the correlator in QPSK works for $2T_b$ resulting in almost the same value of BER. The same holds for OQPSK and $\pi/4$-QPSK.



BPSK

• Power and spectral efficiency

Band limitation of PSK/FSK modulated signal cause amplitude (envelope) variations in a transmitter

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Those variations are subject to nonlinear distortions in PA and result in harmonic or inter-modulation products - "spectral regrowth"

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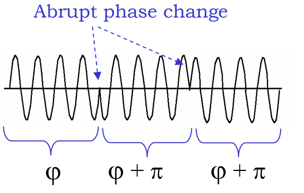
More linear PAs are less efficient, and vice-versa, hence "power efficiency" problem



Also pulse shaping used for narrow band requires a more linear PA

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This can be alleviated with schemes using small phase shift (e.g. $\pi/4$-QPSK), but a trade-off between power efficiency and spectral efficiency exists

Conclusion

- Variety of modulation /detection schemes for transceiver architectures exist
- Quadrature modulation techniques are preferred over simple ones
- Coherent detection more reliable but also more complicated than noncoherent
- Individual design solutions represent different trade-off in performance such as BER, spectral efficiency, and power efficiency